Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

5268134660

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2. [2]

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(b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

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2 A line has equation $y = 2cx + 3$ and a curve has equation $y = cx^2 + 3x - c$, where c is a constant.

Showing all necessary working, determine which of the following statements is correct.

A The line and curve intersect only for a particular set of values of c.

B The line and curve intersect for all values of c.

C The line and curve do not intersect for any values of c. [4]

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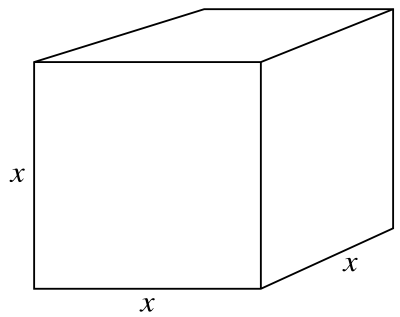

The diagram shows a cubical closed container made of a thin elastic material which is filled with water and frozen. During the freezing process the length, x cm, of each edge of the container increases at the constant rate of 0.01 cm per minute. The volume of the container at time t minutes is V cm^3.

Find the rate of increase of V when $x = 20$. [3]

4 The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = \mathrm{g}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

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(b) Find the equation, $y = \mathrm{h}(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

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(c) State fully the transformation that maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{h}(x)$. [2]

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5 **(a)** Show that the equation

$$4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$$

may be expressed in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers to be found. [3]

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(b) Hence solve the equation $4\sin x + \dfrac{5}{\tan x} + \dfrac{2}{\sin x} = 0$ for $0° \leqslant x \leqslant 360°$. [3]

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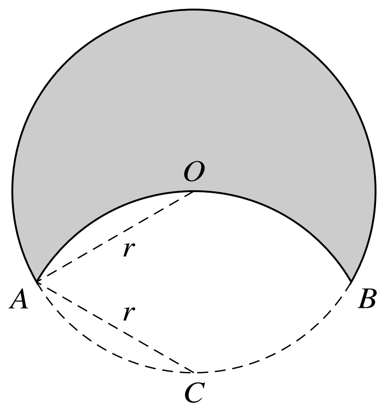

The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O, and the minor arc AOB of a circle, also with radius r but with centre C. The point C lies on the circle with centre O.

(a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k. [1]

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(b) State the perimeter of the shaded motif in terms of π and r. [1]

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(c) Find the area of the shaded motif, giving your answer in terms of π, r and $\sqrt{3}$. [5]

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7 The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression. [7]

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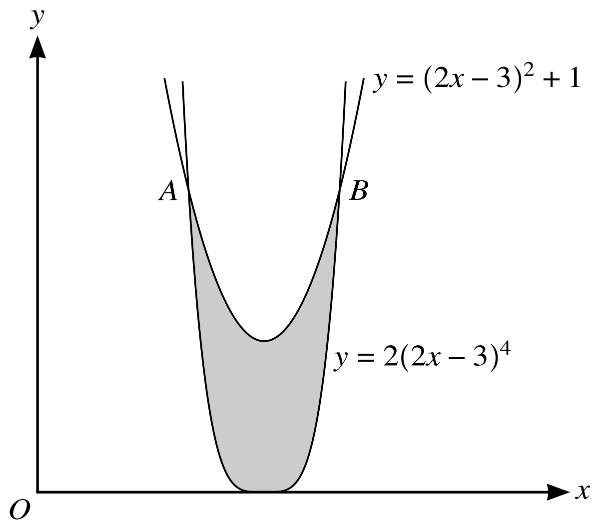

The diagram shows the curves with equations $y = 2(2x - 3)^4$ and $y = (2x - 3)^2 + 1$ meeting at points A and B.

(a) By using the substitution $u = 2x - 3$ find, by calculation, the coordinates of A and B. [4]

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(b) Find the exact area of the shaded region. [5]

9 **(a)** Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

(b) Given that it is possible to form the composite function gf, find the least possible value of p and the greatest possible value of q. [3]

(c) Find an expression for $gf(x)$. [1]

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The function h is defined by $h(x) = 4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for $h^{-1}(x)$. [3]

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10 A curve has a stationary point at $(2, -10)$ and is such that $\frac{d^2y}{dx^2} = 6x$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) Find the equation of the curve. [3]

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(c) Find the coordinates of the other stationary point and determine its nature. [3]

(d) Find the equation of the tangent to the curve at the point where the curve crosses the y-axis. [2]

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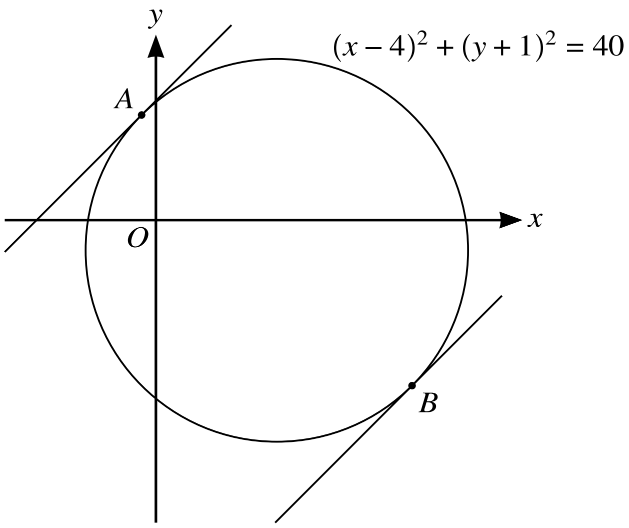

The diagram shows the circle with equation $(x-4)^2 + (y+1)^2 = 40$. Parallel tangents, each with gradient 1, touch the circle at points A and B.

(a) Find the equation of the line AB, giving the answer in the form $y = mx + c$. [3]

(b) Find the coordinates of A, giving each coordinate in surd form. [4]

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(c) Find the equation of the tangent at A, giving the answer in the form $y = mx + c$, where c is in surd form. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.